PREFACE

The consequences of rising concentrations of atmospheric CO_2 are receiving increased attention in the news media as a major contributor to extreme climate change and global warming. For example, from the *New York Times* [12] and restated in Appendix D of this book:

> "Scientific monitors reported that the gas (CO_2) had reached an average daily level that surpassed 400 parts per million (ppm) — just an odometer moment in one sense, but also a sobering reminder that decades of efforts to bring human-produced emissions under control are faltering. The best available evidence suggests the amount of the gas in the air has not been this high for at least three million years, before humans evolved, and scientists believe the rise portends large changes in the climate and the level of the sea."

And from [14], "Climate-related disasters around the world (include): hurricanes and tornadoes, droughts and wildfires, extreme heat waves and equally extreme cold, rising sea levels and floods. Even when people have doubts about the causal relationship of global warming to these episodes, they cannot help being psychologically affected. Of great importance is the growing recognition that the danger encompasses the entire earth and its inhabitants. We are all vulnerable."

However, the CO_2 problem is not well understood quantitatively by a general audience. Although some measures of atmospheric CO_2 accumulation have been reported such as the 400 ppm from the 10 May 2013 *New York Times* noted above, other measures are less well understood, e.g., the increasing rate of CO_2 emissions, the movement of carbon to and from other parts of Earth's environmental system, particularly the oceans with accompanying acidification.

A mathematical model can be particularly informative and helpful for understanding what is happening. We therefore present an introductory global CO_2 model that gives some key numbers, for example, atmospheric CO_2 concentration in ppm and ocean pH as a function of time for the calendar years 1850 (preindustrial) to 2100 (a modest projection into the future).

The model is based on just seven ordinary differential equations (ODEs) and is therefore intended as an introduction to some basic concepts and as a starting point for more detailed studies. The ODEs are carbon balances for seven reservoirs: upper atmosphere, lower atmosphere, long-lived biota, short-lived biota, ocean upper layer, ocean deep layer and marine biosphere.

Basic calculus is the only required mathematical background, e.g., derivatives and integration, so that the model is accessible to high school students as well as beginning college and university students. Specifically, the ODEs define derivatives of the form dy/dt where y is a dimensionless reservoir carbon concentration and t is time as a calendar year over the interval $1850 \leq t \leq 2100$. The ODEs are integrated numerically to give $y(t)$. The solutions are presented numerically and graphically. The essential mathematical concepts are embedded in the fundamental theorem of calculus:

$$\int_{t_0}^{t} (dy/dt)\, dt = y(t) - y(t_0),$$

where $t_0 = 1850$ and $y(t_0)$ is the ODE initial condition (IC). The integration is carried out numerically by library initial-value ODE integrators.

Some background in computer programming would also be helpful, but not essential. The programming of the model is in R[1] and Matlab,[2] two basic, widely used scientific programming systems. Thus, it can be executed on modest computers and is generally

[1] R is an open source scientific programming system that can be downloaded (no cost) from the Internet.

[2] Matlab is a commercial product available from the Mathworks, Natick, MA. Octave is an open source (no cost) alternative that will run the Matlab code with minimal changes.

accessible and usable worldwide. The book could also be used by readers interested in Matlab and/or R programming or a translation of one to the other. This additional use is facilitated by a side-by-side format of sections of R and Matlab code with a comparison of the two.

The model includes ocean chemistry calculations that address acidification with ocean pH as an output, typically ranging from 8.2 to 7.8 (pH decreases with increasing acidity). These calculations illustrate some basic numerical procedures, such as a Newton solver applied to a fourth-order polynomial to calculate ocean pH and spline interpolation to provide additional model outputs. The problem of acidification has important implications for a major food source, deterioration of coral and the associated marine life, as well as ocean CO_2 uptake [9].

A basic global warming component has been added based on CO_2 buildup in the lower atmosphere but since climate change (e.g., warming) is still a controversial and uncertain area, the primary focus is on carbon buildup in the atmosphere and oceans which is being measured quantitatively and is therefore undisputed.

Our intention is to provide a quantitative introduction to the CO_2 problem so that the model is basic and educational, and is not a state-of-the-art research model. We hope that the model serves this educational purpose, and we welcome comments, questions about model implementation and use, and suggestions for improvements (directed to wes1@lehigh.edu).

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